Date 20/04/2020

For (-Mathematics)

Subject - Mathematics

Paper — IV

Chapter - Total differential Equations

By Dr. Ram Anek Yadav Assistant Guest professor, Department of Mathematics, Marwari College, Darbhanga, Mobile No- 9708834223

Problem (1) :- Solve $3x^2dx + 3y^2dy - (x^3+y^3+e^{2z})dz = 0$

Solution: Given differential Equation is

$$3x^2dx + 3y^2dx - (x^3+y^3+e^{2z})dz = 0.$$

Let us take z to be a constant so that $dz = 0$.

The given equation reduces to

$$3x^2dx + 3y^2dy = 0$$

Integrating we get

$$\int 3x^2dx + \int 3y^2dy = \int 0$$

$$\Rightarrow \cancel{3}\frac{x^3}{\cancel{3}} + \cancel{3}\frac{y^3}{\cancel{3}} = \text{Constant} = \phi \text{ (say)}$$

$$\Rightarrow x^3 + y^3 = \text{Constant} = \phi \quad (1)$$

where ϕ may be regarded as function of z.

Differentiating (1) w.r.t. x, y & z we get

$$3x^2dx + 3y^2dy = d(\phi)$$

Comparing with the given equation, we get $d\phi = (x^3+y^3+e^{2z})dz$

Date 20/04/2020
For B.Sc. Part-2 , paper IV
(Honours & Sub.)
Subject- Mathematics
By Dr. Ram Anek Yadav, Deptt of Mathematics
, MarWari College, Darbhanga

$$\Rightarrow \frac{d\varphi}{dz} = x^3+y^3+e^{2z} = \varphi + e^{2z} \quad [\because x^3+y^3=\varphi]$$

$$\Rightarrow \frac{d\varphi}{dz} = \varphi + e^{2z}$$

$$\Rightarrow \frac{d\varphi}{dz} - \varphi = e^{2z}$$

which is linear and its I.F $= e^{\int -1\,dz} = e^{-x} = e^{-z}$

Hence the solution is

$$\varphi . e^{-z} = \int e^{2z} . e^{-z}\, dz = \int e^{z} dz = e^{z} + c$$

$$\Rightarrow \varphi = \frac{e^{z}+c}{e^{-z}}$$

$$= (e^{z}+c)\, e^{z}$$

$$= e^{z} \times e^{z} + c e^{z} = e^{2z} + c . e^{z}$$

i.e. $\varphi = e^{2z} + c . e^{z}$

which is required solution.